

# Discussion and Reply

## Comment on “Frequency-domain Green’s functions for radar waves in heterogeneous 2.5D media”

(K. J. Ellefsen, D. Croizé, A. T. Mazzella, and J. R. McKenna, 2009, *GEOPHYSICS*, 74, no. 3, J13–J22)

Discussion by Juan D. Bulnes<sup>1</sup>, Luis A. Peche<sup>1</sup>, and Jandyr M. Travassos<sup>1</sup>

We call the reader’s attention to a recent paper by Ellefsen et al. (2009) in which the authors use the following equation for the magnetic field  $\mathbf{H}$  expressed in the frequency domain:

$$\nabla \times \left( \frac{1}{Y} \nabla \times \mathbf{H}(\mathbf{r}, \omega) \right) + Z\mathbf{H}(\mathbf{r}, \omega) = -\mathbf{J}_M(\mathbf{r}, \omega) \quad (1)$$

(this is equation 11 in Ellefsen et al., 2009, p. J19). That equation and the commentaries in the sequel are traceable to confusion about the use of the magnetic current density  $\mathbf{J}_M$ , an artificial concept.

In the paper, the authors solve an inhomogeneous vector wave equation for the electric field, which is mathematically identical to equation 1,

$$\nabla \times \left( \frac{1}{Z} \nabla \times \mathbf{E}(\mathbf{r}, \omega) \right) + Y\mathbf{E}(\mathbf{r}, \omega) = -\mathbf{J}(\mathbf{r}, \omega), \quad (2)$$

where  $\mathbf{J}(\mathbf{r}, \omega)$  is the electric current density expressed in the frequency domain. The above equation (equation 2 in Ellefsen et al., 2009, p. J14) is solved using the finite-difference method, as described on pages J14 and J15. Up to this point, the methodology is correct in the context of the problem being analyzed.

Notwithstanding, a flaw arises when the authors claim that a solution for equation 1 can be written by substituting variables accordingly in the solution for equation 2, a procedure based on the similarity of the two equations. That is correct as long as one restricts to the mathematical form of the equations, but equation 1 has a magnetic current term that lacks physical or, for that matter, geophysical meaning.

A magnetic current source does not exist in the equations of classical electrodynamics (e.g., Feynman and Leighton, 1964). Some au-

thors, however, use the concept of magnetic current only as a mathematical artifice based on the symmetry of the *extended* Maxwell’s equations in helping to tackle involved problems by the correspondence between magnetic and electrical parameters (Fiódorov, 1982). In this way, the authors solve the inhomogeneous equation 2 claiming a solution to equation 1 can be found accordingly, but they do not associate that to any physically realizable problem. For instance, this is the case of the well-known “elementary magnetic vibrator” (a fictitious system) and the “slit oscillator” (a real system, in which there is a slit in an ideal metal plate). In that case, a direct correspondence is established between the magnetic current (a fictitious entity) obtained from the magnetic current density  $\mathbf{J}_M$ , and the difference of potential applied between the edges of the slit (a physical entity).

We claim that using the above-cited procedure to solve equation 1 “to simulate electromagnetic waves generated by a magnetic dipole,” as written in the paper (Ellefsen et al., 2009, p. J19), has only a mathematical sense, not a physical one. To conclude, the term  $\mathbf{J}_M$  has never been used to establish a correspondence with any physically realizable situation; equation 1 has to be considered as a homogeneous one.

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## Reply to the discussion

Discussion by Karl J. Ellefsen<sup>2</sup>, Delphine Croizé<sup>3</sup>, Aldo T. Mazzella<sup>4</sup>, and Jason R. McKenna<sup>5</sup>

We wish to address the concerns raised by Bulnes et al. regarding the magnetic dipole and the associated magnetic current density. These entities are mathematical constructs for calculating electromagnetic fields. The magnetic dipole and the magnetic current can represent, for example, an electric current loop (Wait, 1982, p. 101–139); the fields calculated for such a current loop are used to process and interpret data from electromagnetic surveys (Fitterman and Labson, 2005, p. 301–355).

## REFERENCES

- Fitterman, D. V., and V. F. Labson, 2005, Electromagnetic induction methods for environmental problems, in D. K. Butler, ed., *Near-surface geophysics*: SEG Investigations in Geophysics No., 13, 301–355.  
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